Summer Project on Condensed Matter Physics - IISc, Bangalore 2023

Lecture 5: Special Lecture: Conductance

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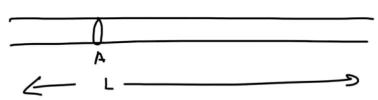
This lecture dealt with electronic transport in mesoscopic systems, in particular dealing with Landauer conductance.

1 Very Small Systems

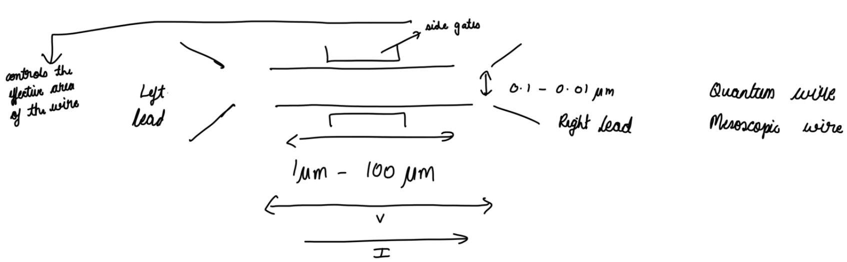
Roughly speaking, we can have a few regimes depending on the size of the system we are talking about.

$$\text{microscopic} \sim 1 \text{ Å - } 100\text{Å}$$
$$\text{mesoscopic} \sim 100 \text{ Å - } 100\ \mu m$$
$$\text{macroscopic} \sim 1\ m$$

In this lecture, we will be talking about mesoscopic systems, in particular very small wires and will check their conductance.
Consider a classical wire (which just means a macroscopic wire):
If the resistivity of the material is ρ, then the resistance of a wire is $R = \frac{\rho L}{A}$, where L is the

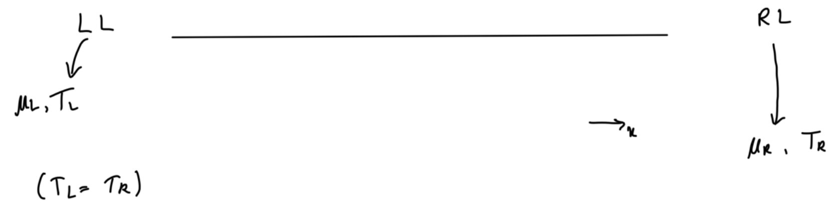


length of the wire and A is the cross-section of the wire. The current flowing through it when a potential difference V is applied across its ends is $I = \frac{V}{R}$. Alternate to its resistance, we can define its conductance as $G = \frac{1}{R} = \frac{I}{V} = \frac{A}{\rho L}$.$\rightarrow$The question we should ask next is whether all the above relations hold for a system which is really small.



2 Landauer Conductance

To start off with the discussion of conductance for very small systems, let us consider a simple model system. The system is a one-dimensional wire with particular chemical potentials on its two ends, μ_L on the left and μ_R on the right. Consider the temperature on the two ends to be same, i.e., $T_L = T_R$.



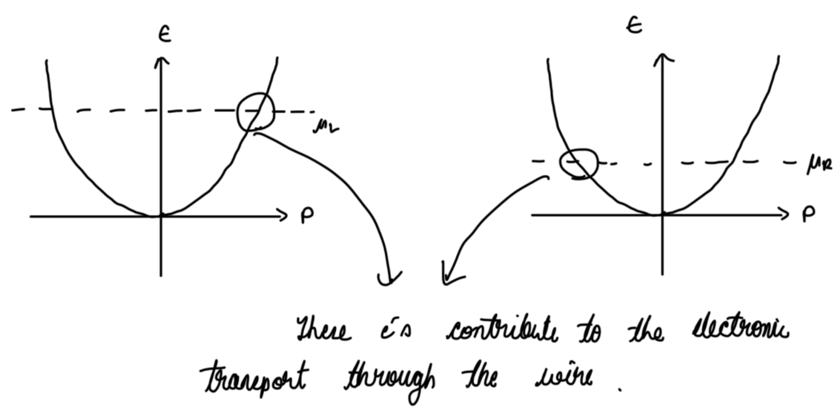
At the leads, we can write the Hamiltonian as:

$$H_{LL} = \frac{\vec{p}_x^2}{2m} - \mu_L \tag{1}$$

for the left lead; and

$$H_{RR} = \frac{\vec{p}_x^2}{2m} - \mu_R \tag{2}$$

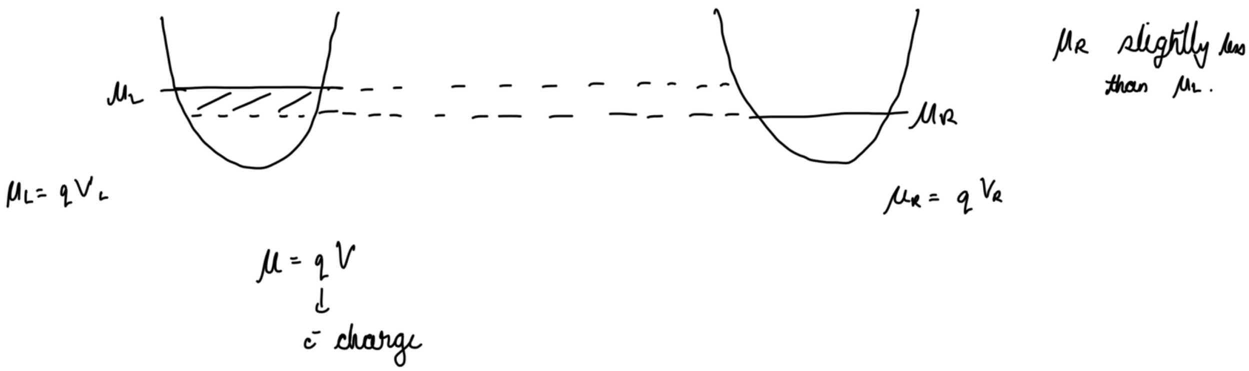
for the right lead.



The group velocity is defined as:

$$v_{group} = \frac{d\epsilon}{dp} \tag{3}$$

The net current I will be 0 if $\mu_L = \mu_R$, which is not too interesting.
Now, if we have μ_R is slightly less than μ_L, then things get slightly more interesting.



The current is defined as:

$$I = \frac{q}{L}\int_{\mu_R}^{\mu_L} d\epsilon \rho(\epsilon) v(\epsilon) \text{ (here } \rho \text{ is density of states and not resistivity)} \tag{4}$$

In general, for E_p having some dependance on p, we can the density of states as:

$$\begin{aligned}\rho(E) &= \int \frac{dp}{2\pi\hbar/L}\delta(E - E_p) \\ &= \frac{L}{2\pi\hbar}\frac{1}{\left(\frac{dE_p}{dp}\right)_{E_p=E}}\end{aligned}$$

$$\implies \rho(E) = \frac{L}{2\pi\hbar}\frac{1}{v_{group}(E_p)} \tag{5}$$

Thus, we get

$$\begin{aligned}\therefore I &= \frac{q}{L}\int_{\mu_R}^{\mu_L} d\epsilon \frac{L}{2\pi\hbar} = \frac{q}{2\pi\hbar}(\mu_L - \mu_R \\ &= \frac{q^2}{2\pi\hbar}(V_L - V_R) \\ &= \frac{q^2}{h}(V_L - V_R) \\ \implies G &= \frac{I}{V_L - V_R} = \frac{q^2}{h}\end{aligned}$$

$$\therefore G = \frac{q^2}{h} \tag{6}$$

For electrons, we have 2 possibles spins, and we get

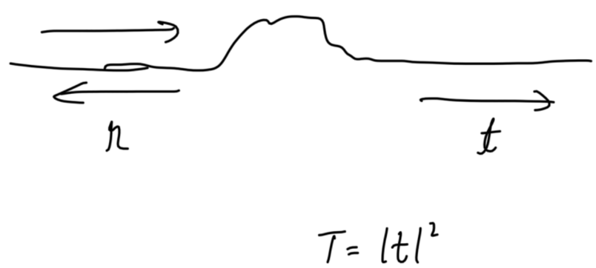
$$G_0 = \frac{2e^2}{h}$$

with

$$\frac{h}{2e^2} = 12.9k\Omega$$

Here, we should note that the result we have obtained is kind of weird if we just consider the system with a clear thought. We considered a "clean" wire in which the electrons move ballistically. We should expect all electrons to go straight through.
But our result shows that the wire has some resistance. The question is from where are we getting this resistance?
$\rightarrow$This resistance is actually coming from the dissipation of energy near the boundary of the wire and the leads - also called contact resistance.

Till now we considered that there is no actual potential. But the case with $V(x) \neq 0$ is also not so complicated.



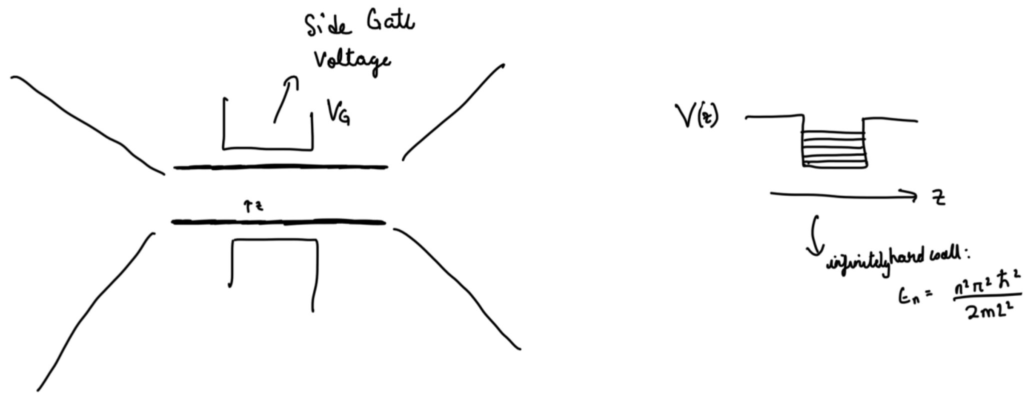
Knowing that for a given $V(x)$, we have reflected and transmitted amplitudes r and t, we can write the transmittance as $T = |t|^2$.
In such a case, the Landauer conductance just changes to

$$G = \frac{2e^2}{h}T \tag{7}$$

3 Wire with Thickness

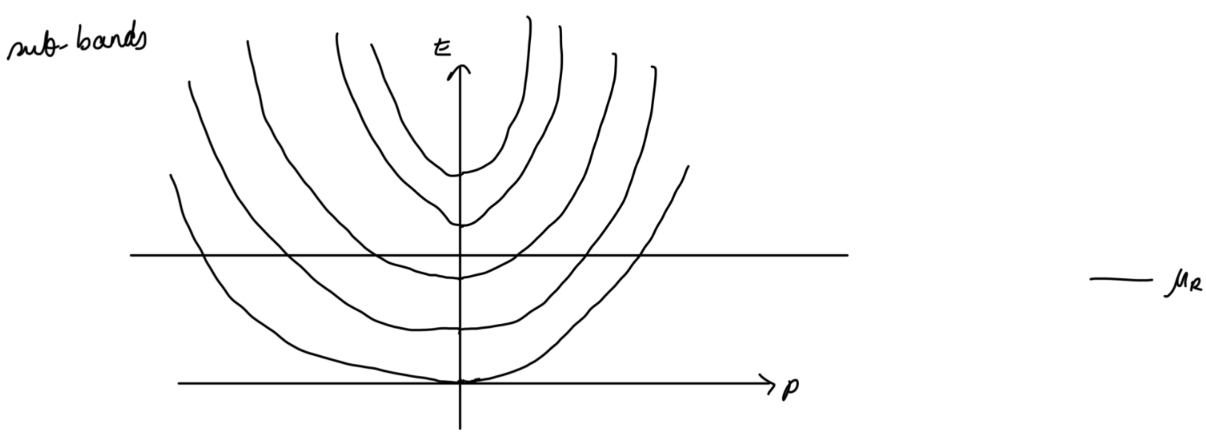
We did the analysis for a 1-D wire till now. Now consider a wire with a certain thickness. The Hamiltonian can be written as:

$$H = \frac{\vec{p}_x^2}{2m} + V(y, z) \tag{8}$$



Considering the side gate voltage to be strong enough, we can approximately write the energy levels of the electrons in the wire as:

$$E(p, n) = \frac{p^2}{2m} + \frac{n^2\pi^2\hbar^2}{2mW^2} \tag{9}$$

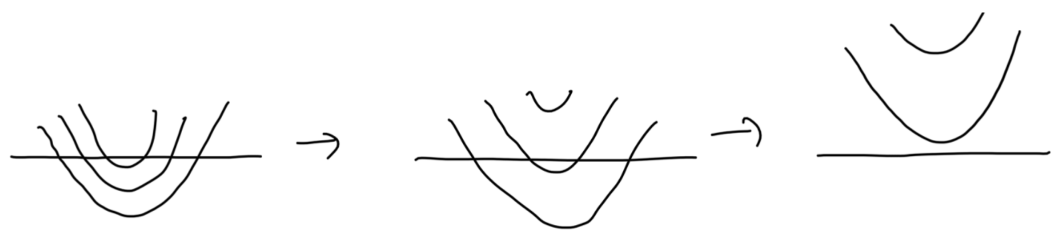


The second term in the right hand side only displaces the energy spectrum along the y-axis in the E vs. p plot.

For the energy spectrum shown above, we get conductance as

$$G = \frac{2e^2}{h} 3$$

As negative V is increased, W decreases.



For different sub-bands with different transmission coefficients:

$$G = \frac{2e^2}{\hbar} \sum_n T_n(E) \tag{10}$$

The mesoscopic nature i.e., ballistic motion of electrons is granted when mean free path of electrons $>>$ length of wire.

> In all this, Buttiker's contribution was the generalised measurement of current/voltage: four-probe measurement.